Fine-grained Parameterized Algorithms

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Problem set 3: Iterative Compression and Randomization

Overview: With this problem set, you can train developping FPT algorithms using iterative compression and randomization.

Instructions: For each skill, select **exactly one** problem below and submit your solution in Moodle; in your submission, make sure to repeat the problem that you are solving. The problems are roughly ordered by difficulty, choose a problem that you find non-trivial and interesting. (You are of course welcome to try the other problems as well and ask us for feedback.)

Note: On this problem set, we write *one-sided bounded-error randomized algorithm* for any randomized algorithm that has a one-sided error probablitity of at most $1/2$.

‼ **Skill-3a.** **Iterative Compression:** *I can design FPT algorithms using iterative compression.* (For an example, see Section 4.1 and 4.2 in Cygan et al.)

3.1 3-Hitting Set. Obtain an algorithm for 3-*Hitting Set* running in time $2.4656^k n^{O(1)}$ using iterative compression. Generalize this algorithm to obtain an algorithm for d-*Hitting Set* running in time

$$((d-1)+0.4656)^k n^{O(1)} .$$

3.2 Independent feedback vertex set. A set $X \subseteq V(G)$ of an undirected graph G is called an *independent feedback vertex set* if $G[X]$ is independent and $G - X$ is acyclic. In the *Independent Feedback Vertex Set* problem, we are given as input a graph G and a positive integer k, and the objective is to decide whether G has an independent feedback vertex set of size at most k. Show that this problem is fixed-parameter tractable by obtaining an algorithm running in time $5^k n^{O(1)}$ using iterative compression.

‼ **Skill-3b.** **Color coding:** *I can design randomized FPT algorithms by applying the color coding technique.* (For more context, see Chapter 5 in Cygan et al.)

3.3 Triangle Packing. In the *Triangle Packing* problem, we are given an undirected graph G and a positive integer k, and the objective is to test whether G has k vertex-disjoint triangles. Use color coding to design a one-sided bounded-error randomized algorithm for this problem with running time $2^{O(k)} n^{O(1)}$.

3.4 Tree Subgraph Isomorphism. In the *Tree Subgraph Isomorphism* problem, we are given an undirected graph G and a tree T on k vertices, and the objective is to decide whether there exists a subgraph in G that is isomorphic to T. Use color coding to design a one-sided bounded-error randomized algorithm for this problem with running time $2^{O(k)} n^{O(1)}$.

Additional problems. The following problems help you gain a deeper understanding of the course material. Problem 3.5 combines your knowledge of bounded search trees from last week with the technique of probability amplification.

easy **3.5 Vertex-Cover Probability Amplification.** Design a randomized polynomial-time algorithm $\mathbb{A}$ that, given a graph G and a positive integer k, outputs 0 with probability 1 if G has no vertex cover of size k and outputs 1 with probability $\geq 2^{-k}$ if G has a vertex cover of size k. Then use $\mathbb{A}$ to obtain a randomized algorithm for Vertex Cover running in time $2^{O(k)}n^{O(1)}$ that succeeds in the positive case with probability $\geq 1/2$. Please formally prove the bound on the success probability.

3.6 Challenging graph problem. Consider the following problem: Given an undirected graph G and positive integers k and q, find a set X of at most k vertices such that $G - X$ has at least two components of size at least q.

a) Show that this problem can be solved in time $2^{O(q+k)}n^{O(1)}$ by a one-sided bounded-error randomized algorithm.

b) Assuming $q > k$, show that the problem can be solved in time $q^{O(k)}n^{O(1)}$ by a one-sided bounded-error randomized algorithm.